Satellite Navigation

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LEO satellite and the observed GNSS satellite, commonly referred to as the observed-minus-calculated (O-C) residual.

e linearized error equation for the carrier phase corresponding to the observation Eq. (2) is expressed as

where $_{i1}$

covariance matrix denoted as $\boldsymbol{\Sigma}_{\overline{X}_k^i}$. e corresponding weight matrix is denoted as $\boldsymbol{P}_{\overline{X}_k^i} = \boldsymbol{\Sigma}_{\overline{X}_k^i}^{-1}$.

Stepwise OD based on spaceborne GNSS observations and ISL ranges

e first step of autonomous OD is based on spaceborne GNSS observations. Let the orbital parameter vector to be estimated for each LEO satellite i at epoch k is denoted as $\mathring{X}_k^i$, with its covariance matrix $\sum_{\mathring{X}_k^i}$. e least squares estimator is expressed as:

$$\delta\mathring{X}_k^i = \left[\left(A_{Gk}^i\right)^{\mathsf{T}} P_{Gk}^i A_{Gk}^i\right]^{-1}\left(A_{Gk}^i\right)^{\mathsf{T}} P_{Gk}^i l_{Gk}^i \qquad (18)$$

and the updated state vector is:

$$\mathring{X}_k^i = X_k^{0i} + \delta\mathring{X}_k^i \qquad (19)$$

where X_k^{0i} is the initial approximate state parameter vector of the satellite i, without any prior information, and $P_{Gk}^i = \boldsymbol{\Sigma}_{l_{Gk}^i}^{-1}$ is the weight matrix of l_{Gk}^i. e posterior covariance matrix of the estimated orbit parameters using the kinematic OD approach is given by:

$$\sum_{\mathring{X}_k^i} = \left[(A_{Gk}^i)^T P_{Gk}^i A_{Gk}^i\right]^{-1}\hat{\sigma}_0^2 \qquad (20)$$

where $\widehat{\sigma}_0^2$ is the estimated variance factor.

If the predicted dynamic orbit parameter vector $\overline{X}_k^i$ for each satellite is employed, the estimator for the orbit parameter vector is given by:

$$\hat{X}_k^i = \overline{X}_k^i + \left[P_{\overline{X}_k}^i + (A_{Gk}^i)^{\mathsf{T}} P_{Gk}^i A_{Gk}^i\right]^{-1}\left(A_{Gk}^i\right)^{\mathsf{T}} P_{Gk}^i l_{Gk}^i \qquad (21)$$

where $l\ \ _k$ is expressed as:

$$l_{Gk}^i = L_{Gk}^i - A_{Gk}^i \overline{X}_k^i \qquad (22)$$

e covariance matrix of the estimated orbit parameters is as follows (Yang et al., 2001):

$$\sum_{\hat{\mathbf{X}}_k^i} = \left[\mathbf{P}_{\overline{X}_k}^i + (\mathbf{A}_{Gk}^i)^{\mathrm{T}} \mathbf{P}_{Gk}^i \mathbf{A}_{Gk}^i\right]^{-1}\hat{\sigma}_0^2 \qquad (23)$$

Similarly, the orbit parameter estimates $\widehat{X}_k^j$ for the j-th satellite along with its covariance matrix $\boldsymbol{\Sigma}_{\widehat{X}_k^j}$ can be obtained using the same approach.

An alternative recursive estimator of the orbit parameters is (Yang and Gao 2006; Yang et al., 2001):

$$\hat{X}\ \ = \overline{X}\ \ + \boldsymbol{\Sigma}_{\overline{X}}\ \left(A\ \ \right)^{\mathrm{T}}\left[A\ \ \ \sum_{\overline{X}}\ \ = \left(A\ \ \right)^{\mathrm{T}} + \sum\ \ \right]^{-1} l \qquad (24)$$

And the posterior covariance matrix is followed:

e second step of the OD involves using the ISL range measurements between the satellite i and j to calculate the orbit parameter corrections. e corresponding range error equation is:

where $\delta X_k^{ij} = \begin{matrix} \delta X_k^i \\ \delta X_k^j \end{matrix}$ is the correction vector for $\widehat{X}_k^{ij}$, which is expressed as $\widehat{X}_k^{ij} = \begin{pmatrix} \widehat{X}_k^i \\ \widehat{X}_k^j \end{pmatrix}$ and estimated from the first step OD, and l_{sk}^{ij} is the vector of (O-C), with the expression as:

where $A_{sk}^{ij} = [\,A_{sk}^i\ \ -A_{sk}^j\,]$ denotes the design matrix and L_{Gk}^{ij} is the observation vector.

According to the principle of sequential least squares, the correction vector estimator for the orbit parameters is given by:

e estimator can be equivalently expressed as:

It is important to note that the matrix inversion in

Obviously, when any additional ISL range observation is used in the OD, the posterior covariance matrix of the orbital parameter estimates is improved as

It should be pointed that the orbit determination at the second step is easy in computation without any ill-posed problem, because the a priori orbit parameters are estimated with covariance matrix at the first step.

Stepwise OD with ISL distance constraints

We know that the microwave ranging accuracy between two satellites is at centimeter level, and the laser ranging accuracy at millimeter level, and the accuracy of GNSS-

With the support of spaceborne GNSS observations and ISL range observations, the contribution of the dynamic model information can be adaptively adjusted. Assuming that the orbit parameter vector $\overline{\boldsymbol{X}}_k^i$ of the $\boldsymbol{i}$-th satellite at epoch k can be obtained by dynamic state error Eq. (15), the adaptive recursive solution of the satellite orbit parameters based on the dynamic model information and the on board GNSS observations can be expressed as fol

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It should be noted that the orbit parameters of the simulated LEO satellites, with Walker constellation configuration and the symmetric measurement topology, are little influenced by longer time variation. The main time-dependent influencing factors are the Earth's gravitational field and the relative measurement topology, which are characterized by a 24-h periodicity. Therefore, we only simulated the arc length of 24-h satellite measurements for the demonstration of the proposed OD strategies.

Furthermore, when the Earth gravity, atmospheric drag and solar radiation pressure perturbation effects are reasonably considered, the residual dynamical model error caused by the orbit altitude is less than 10% experientially. In our simulation, the same magnitude of the dynamic model error is used, and the orbit altitude does not significantly affect the conclusions of this paper.

Simulated observation data

Usually, the carrier phase and pseudorange should be simulated for the spaceborne GNSS, especially the accuracy of the carrier phase observation is much higher than that of the pseudorange observation. However, the pseudorange observations are usually employed in the actual orbit determination in real time for a very large satellite constellation, and the accuracy of the OD using the pseudorange observation meets the user requirements. Therefore, the pseudorange observations of the spaceborne GNSS receivers and the ISL range observations between LEO satellites are simulated to simplify the verification calculation. We simply assume that each LEO satellite is connected to four neighboring satellites, namely the front and back satellites in the same orbital plane, and the right and left satellites in the adjacent orbital planes. The simulated pseudorange noise of the spaceborne GNSS observation is 0.3 m, with a cutoff elevation angle of 15 degrees; the simulated range noise of the ISL ranges between the LEO satellites is 0.05 m, the sample interval is 30 s, and the simulation time span is one day. The main errors affecting the OD of LEO satellites are the GNSS ephemeris errors and clock bias. The ephemeris random error of the BDS satellites is assumed to be 1.5 m, while the clock random error of the GNSS receiver is assumed to be 0.5 m. Considering that the clock rate difference introduced by relativity is accurately eliminated through rigorous formulation, and the ionosphere effects are eliminated by the dual frequency difference observations, the residual error is less than 0.01 m,

Table 1 [illegible]

Num	*a*(km)	*e*	*i*	*Ω*	*ω*	*M*	Num	*a*(km)	*e*	*i*	Ω	ω	*M*
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
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Table 2 [illegible]

Perturbation Force	Model (Montenbruck & Gill (2000))
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

erefore, we can ignore the above two types of errors in the simulation observations.

Results and analysis

Scheme 1: e kinematic OD approach is performed using the spaceborne GNSS pseudorange observations. e estimated parameters include the position vector and clock bias of the LEO satellite. e statistical results of the RMSE of the estimated orbit parameters are shown in Fig. 1a

Scheme 2: e stepwise kinematic OD approach is performed using both the spaceborne GNSS pseudorange and the ISL range observations. e estimated parameters are the same as those of the Scheme 1. e RMSE of the estimated orbit parameters is shown in Fig. 1b

Scheme 3: e stepwise kinematic OD approach, with ISL range as a constraint, is performed. e estimated parameters are the same as those of Scheme 1. e RMSE of the estimated orbitparameters is shown in Fig. 1c

Scheme 4: e dynamic OD approach is performed. e dynamic reference orbit parameters are generated by adding some simulated systematic errors to the standard orbital parameters of each LEO satellite, which is then used to calculate the predicted state based on the transition equation of the LEO satellite. e parameters of the LEO satellite orbit to be estimated include the position

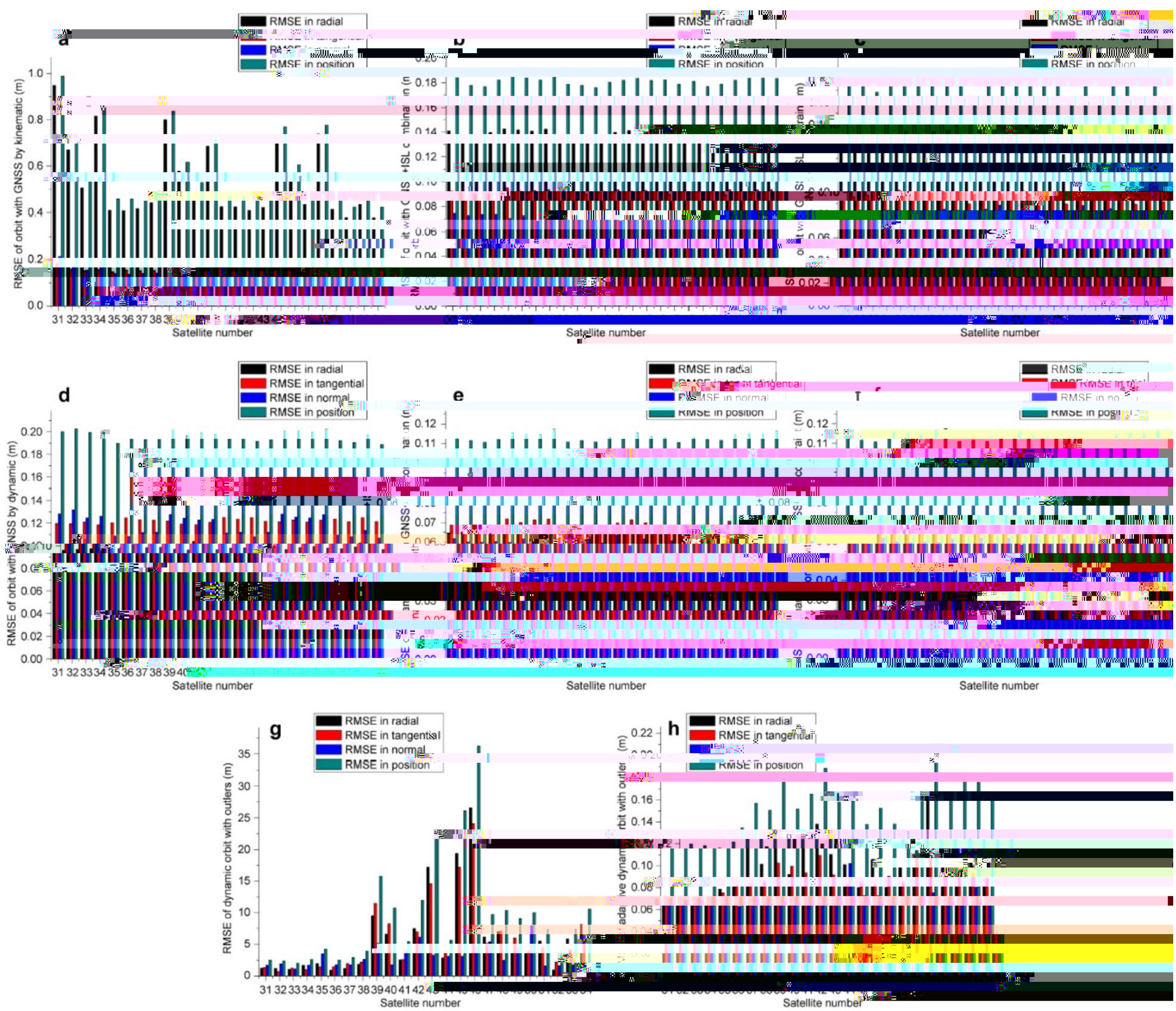

Fig. 1 (a) (b) (c) (d) (e) (f) (g) (h)

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vector, velocity vector, and clock o set. e RMSE of the estimated orbit parameters is shown in Fig. 1d

Scheme 5: e stepwise dynamic OD approach is per-
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OD or dynamic OD with the ISL range constraints, and thus the computational burden is significantly reduced.

(5) Significant disturbances in the dynamic model can lead to the substantial errors in the estimated

as constraints in the stepwise OD strategy. Both theoretical derivations and calculation results indicate that the integrated and stepwise OD approaches are nearly equivalent. e latter can separately estimate the orbit parameters for each LEO satellite in parallel. In addition, the ISL range observations can significantly improve the accuracy of the estimated orbit parameters. More importantly, the adaptive stepwise OD mode with the adaptive factor acting on the covariance matrix of the dynamic model information can e ectively control the dynamic model error influence.

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